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Percolative phase transition in a disordered Ising model with finite disorder correlation length

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Abstract. We discuss the phase transition in an Ising model with correlated disorder. Two parameters describe the disorder: its variance and its finite correlation lengthscale. We show that in this model, depending on the disorder parameters, one of two qualitatively different scenarios for the transition applies. The first is a transition driven by thermal fluctuations around a spatially homogeneous ground state. This is also found in systems with uncorrelated disorder. The second scenario is a percolative one: locally ordered regions grow in the paramagnetic phase and form an infinite cluster at the critical temperature. In contrast to the first scenario, thermal fluctuations now occur around an inhomogeneous ground state. The dominating lengthscale is not the correlation length of thermal fluctuations but the connectivity length of ordered regions. Based on a discussion of the role of thermal fluctuations in the percolative scenario we identify the parameter ranges in which the different scenarios apply.

1. Introduction

The renormalization group (RG) has become a standard tool in the theory of critical phenomena. It provides very successful quantitative predictions for universal quantities such as critical exponents and amplitude ratios which are accessible to experiments and computer simulations. At the same time it is based on a simple physical picture which also explains the phenomenon of universality: the partition function can be written as a functional integral over thermal fluctuations around a spatially homogeneous ground state. The correlation length of these fluctuations diverges as the critical temperature is approached. Consequently all other scales of the system can be neglected. Microscopically different systems show the same critical behaviour which depends only on very few characteristics of the system, such as dimensionality and symmetry of the system and its order parameter.

This concept has also been applied successfully to disordered systems. Usually the disorder is averaged over right at the beginning of the theoretical analysis, for example using the replica trick. Then the same picture as in the ordered case is recovered, including fluctuations around a spatially homogeneous ground state, and the RG procedure can be used along the same lines as for ordered systems. This is even possible for some types of disorder with scaling invariant, infinite-range disorder correlations [1–4]. Systems for which this procedure is successful are often called 'weakly' disordered.

For disordered systems there are, however, only much less reliable numerical and experimental results that can be compared with RG predictions. This is partly due to difficulties in finding adequate mathematical models for particular physical disordered systems. Generally, the frontier between 'weak' and 'strong' disorder can only be

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determined by the investigation of a particular physical problem. In several cases disorder can be characterized by a set of parameters, and the critical behaviour of the disordered system is very similar to the pure one where these parameters are zero. This kind of disorder may well be called 'weak'. But there are also several counterexamples where the critical behaviour is changed qualitatively even for arbitrarily small nonzero values of these parameters, e.g. the two-dimensional random field Ising model or the Anderson transition in two dimensions.

In this paper we discuss a simple model in which the disorder is described by two parameters, the variance of the disorder (which is often considered the only relevant disorder parameter) and an additional (finite) correlation lengthscale of the disorder. We will show that in this model two qualitatively different scenarios of the transition appear, a percolative and a homogeneous one, depending on the ratio of the two parameters of the model. This will allow a general discussion of the two scenarios and of a possible criterion to discern the two.

In the percolative scenario, even above T_c large 'locally ordered' regions appear where the order parameter fluctuates around a nonzero value (however, the *average* value of the order parameter is still zero above T_c because both signs of the local order parameter appear with the same probability in such a finite region). This leads to a picture of the transition which is qualitatively different from the homogeneous one usually connected with the RG approach: now there is a spatially inhomogeneous, temperature-dependent 'ground state' of the system. Actually, there is a multiplicity of such states differing from each other by the sign of the order parameter in disconnected ordered regions. A new relevant lengthscale appears in addition to the correlation length of thermal fluctuations around the inhomogeneous ground state: the size of locally ordered regions, which diverges as the connectivity length of the percolation problem for the growing locally ordered regions. We will refer to the two scenarios as 'homogeneous' resp. 'percolative' because of this difference in the spatial structure of the ground state.

We will illustrate these points in some detail for a special version of this model which permits an explicit analysis of the competition between statistical geometric disorder fluctuations and thermal fluctuations of the order parameter near the phase transition. In particular we will discuss the topological aspects of this competition. This will illustrate why the usual RG treatment (with its approximation of δ-correlated disorder) fails for the corresponding values of the model parameters.

This paper is organized as follows. In section 2 we review some aspects of the standard RG treatment of disordered systems that limit this approach to the case of weak, uncorrelated disorder, as well as some approaches used to overcome this limitation for strongly disordered systems.

In section 3 we introduce a general model of disorder characterized by two parameters. It is worth noting that disorder in this model is not 'strong' as in some of the models mentioned in section 2—the variance of the disorder is a small parameter, and the disorder correlation length is finite, so no new diverging lengthscale is put in 'by hand'. Earlier treatments of disordered systems via a one-instanton approximation for the disorder-averaged Hamiltonian are briefly reviewed and applied to our model; the limitation of this approach to a temperature range away from the critical point is discussed.

In section 4 we analyse in detail a specialized version of the model introduced in section 3; this model clearly exhibits a percolative transition into an inhomogeneously ordered phase. The role of thermal fluctuations must be analysed together with the influence of the temperature-independent, topological properties of the disorder; this is discussed in section 4.2. At the end of section 4 we briefly discuss the generalization of these results to

other versions of the two parameter model, and mention some reasons for the failure of the Harris criterion to detect the deviation of the system's behaviour from the ordinary scenario dominated by thermal fluctuations. In the final section, 5, we summarize the main results and show how they contribute to some current debates mentioned in section 2, including the role of multiple ground states and replica symmetry breaking (RSB). We will also discuss briefly the relevance of our results for the explanation of various recent experiments in which two lengthscales were observed simultaneously near the magnetic phase transition in Holmium, Terbium and even perovskites with a weakly first-order transition.

2. Limitations of the conventional RG approach

A basic assumption of the conventional RG method is that the (thermal) correlation length ξ, which describes the characteristic lengthscale of thermal fluctuations of the order parameter, grows without limit in the vicinity of the critical point. At first sight this seems to imply that disorder with any (finite) correlations could effectively be modelled with a simple δ-function for the two-point correlation function of the disorder. In addition, the formal irrelevance of higher cumulants of the quenched disorder probability distribution within the framework of the commonly used $\varepsilon = 4-d$ expansion, which holds at least in the limit $\varepsilon \to 0$, has led to the conclusion that the critical behaviour of 'weakly' disordered systems can be described approximating the disorder as Gaussian and δ-correlated [5].

So investigations of the critical behaviour of 'weakly' disordered systems are usually based on a very simple model Hamiltonian [6]

$$H = H_{\mathrm{GL}}[\varphi] + \int \mathrm{d}^d r \, \{\Delta\tau(\boldsymbol{r})\varphi^2(\boldsymbol{r})\} \tag{1}$$

where the Ginzburg–Landau Hamiltonian for the pure system is given by

$$H_{\mathrm{GL}}[\varphi] = \int \mathrm{d}^d r \, \left\{ \frac{1}{2}(\nabla\varphi)^2 + \frac{r_0}{2}\varphi^2 + \frac{1}{4}\lambda_0\varphi^4 \right\}. \tag{2}$$

The second term in (1) describes the disorder. $\Delta\tau(\boldsymbol{r})$ is sometimes referred to as a random variation of the 'local reduced temperature' $\tau(\boldsymbol{r}) = \frac{T-T_c(\boldsymbol{r})}{T_c^{(0)}} = r_0 + \Delta\tau(\boldsymbol{r})$, because r_0 is connected with the reduced temperature in the undisturbed case. Using this suggestive terminology the local critical temperature then varies as $T_c(\boldsymbol{r}) = T_c^{(0)}(1 - \Delta\tau(\boldsymbol{r}))$. The random variable $\Delta\tau(\boldsymbol{r})$ is usually assumed to be Gaussian and δ-correlated, so

$$\langle\tau(\boldsymbol{r})\rangle = r_0; \langle\Delta\tau(0)\Delta\tau(\boldsymbol{r})\rangle - \langle\Delta\tau(\boldsymbol{r})\rangle^2 = u_0\delta(\boldsymbol{r}). \tag{3}$$

Here u_0 is a measure of the defect 'strength'. In this model it is the only parameter characterizing the disorder. Without loss of generality we can assume that $\langle\Delta\tau(\boldsymbol{r})\rangle = 0$.

Then the standard replica trick is applied to perform the average over disorder configurations, and a translationally invariant effective Hamiltonian is recovered. The further RG treatment is very similar to that of pure systems [6]. The appearance of a new ('disorder') stable fixed point in this treatment is considered as a reliable proof for the validity of the general assumption that the same homogeneous scenario of critical phenomena [7] holds for pure and disordered systems; it can be used to justify the simplified model (1)–(3) in a self-consistent way [8].

Correspondingly it was concluded that the free energy of a wide class of 'weakly' disordered systems has a singularity at a critical temperature $T = T_c$. This T_c may differ from that of the pure system, but the transition will remain 'sharp', not smeared out. Near this point the free energy is expected to display scaling as in the case of pure systems. The

difference in the critical behaviour of pure and disordered systems is limited to changes in the values of the critical exponents which are now determined by a new, stable 'disorder' fixed point. As predicted by the phenomenological Harris criterion [9] and confirmed by explicit RG calculations even these changes only occur if the critical exponent α of the specific heat is positive for the undisturbed model.

The RG results have one strange feature, however: all the RG flow trajectories of the renormalized vertices $\lambda(t), u(t)$ starting inside the 'physical' region $\lambda_0, u_0 > 0$ reach the stable (disorder) fixed point [10]. So even in the case of a large initial disorder vertex, $0 < \lambda_0 \ll u_0$ the critical behaviour should asymptotically be the same as for the conventional 'weak' disorder case $\lambda_0 \gg u_0$.

Of course the range of applicability of (1)–(3) can be restricted to initial values $\lambda_0 \gg u_0$ by an appeal to physical arguments. But if we stay within the formal framework of the RG itself there is no inherent evidence for any nontrivial separatrix in the (λ, u)-plane between 'weakly disordered' systems and others whose critical behaviour is not controlled by the conventional disorder stable fixed point. In some sense the critical behaviour predicted by the standard RG treatment is 'too universal' to be realistic for systems with arbitrary disorder parameter values, including strong disorder.

From a physical point of view it is clear that in the limit of very strong fluctuations of $\tau(\boldsymbol{r})$ the phase transition may differ qualitatively from the homogeneous scenario. In the case of the model (1)–(3) a new transition scenario for strong disorder has been discussed very early in the context of dirty superconductors [11] and dirty magnets or alloys [12].

In both cases it was assumed that in regions where $\tau(\boldsymbol{r})$ is negative, i.e. in the deepest 'wells' of the random 'potential' $\tau(\boldsymbol{r})$, nuclei of the ordered phase may appear even far above the disorder-averaged critical temperature $T_c = \langle T_c(\boldsymbol{r})\rangle$. This is possible if the free energy gained by the formation of such nuclei is much larger than $k_B T_c$. Far above T_c these nuclei are far apart from each other; however, at the edges of the nuclei the order parameter only vanishes exponentially over the thermal correlation lengthscale ξ, and the overlap of the edges of neighbouring nuclei leads to an effective interaction $V(r) = V_0 \exp(-r/\xi)$ between them [13].

This interaction favours nuclei with equal signs of the order parameter, and it has been postulated that interacting nuclei have the same sign of the order parameter if they are a distance $r < \tilde{r}$ apart, where $\tilde{r}$ is given by

$$V(\tilde{r}) = k_B T. \tag{4}$$

This requires a large interaction, $V_0 \gg k_B T_c$, so it can only appear in strongly disordered systems.

If nuclei less than $\tilde{r} = \tilde{r}(T)$ apart have the same sign of the order parameter, the phase transition problem is reduced to the continuous percolation problem [14] for spheres with a radius $\tilde{r}$.

For some types of strongly disordered materials such an approach is quite appropriate. However, these arguments and a condition such as (4) almost completely neglect the role of thermal fluctuations. On the other hand, in 'weakly' disordered models thermal fluctuations play a crucial role—this is the very basis for the homogeneous scenario which allows the standard RG treatment including the replica trick. Therefore one would like to have an explicit criterion which allows us to determine the character of the transition in a given disordered solid—transition into a homogeneous phase driven by thermal fluctuations or percolative transition into an inhomogeneous phase formed by interacting 'nuclei' of the ordered phase.

A first step towards such a criterion was made by Bulaevskii *et al* [15] in an investigation

of a very dirty superconductor near the Anderson localization threshold (where both disorder and thermal fluctuations are strong enough to be relevant). They have shown that in this system isolated nuclei of the ordered phase are stable against thermal fluctuations if $\lambda_0 < \lambda_c = Au_0$ with a prefactor A of order 1. If $\lambda_0 > \lambda_c$ thermal fluctuations are too strong and nuclei of the ordered phase cannot be formed. However, their approach was limited to the temperature range $\tau \gg u_0^2$, so the true nature of the transition could not be established.

Another discussion of the model (1)–(3) with $\lambda_0 \leqslant u_0$ was in [13], where the critical behaviour close to the tricritical point was considered in a more general context not restricted to a particular case of the superconducting transition. There a transition of a percolative type with the formation of isolated 'droplets' of the ordered (low-temperature) phase was found, too, in contrast to that predicted by the standard RG treatment.

However, the reason for this qualitative difference has apparently not been discussed in [13] nor elsewhere. This may partly be due to the amount of additional approximations and assumptions made in [13]. In particular their analysis uses $(\ln(u/\lambda))^{-1}$ as a small parameter. The success of this procedure may depend significantly on their choice of a particular (simplified) model; it is not clear that it will also apply to a general disordered material near the tricritical point.

Quite recently, however, the validity of the standard RG procedure for the model (1)–(3) has been questioned again, [16] even for 'weakly' disordered systems. The existence of many nearly degenerated ground states of the system was discussed as a physical reason for the appearance of RSB terms in the Hamiltonian. This applies even to the model (1)–(3) with 'weak' disorder (for 'strongly' disordered systems solutions with RSB have been discussed before [17–20]). Under several assumptions a mechanism for the appearance of RSB interactions in the model (1)–(3) was demonstrated and the corresponding 1-step RSB solution of the RG equations was determined. But the stability of the RSB solutions (including multistep solutions) is very questionable [16]. The same may be said about the validity of the particular mechanism giving rise to RSB terms in the effective Hamiltonian and their explicit form. So although the general ideas presented in [16] seem to be reasonable and very attractive, the whole problem remains unsettled.

3. Two-parameter model of correlated disorder

The model we consider is a variation of the standard one in (1)–(3), but (3) is generalized to a nontrivial correlation function for the Gaussian disorder,

$$\langle \tau(\boldsymbol{r}) \rangle = r_0;\ \langle \Delta\tau(0)\Delta\tau(\boldsymbol{r}) \rangle - \langle \Delta\tau(\boldsymbol{r}) \rangle^2 = \tau_1^2 G(r/R_0) \tag{5}$$

with a correlation function $G(x) \approx 1$ for $x \leqslant 1$ and $G(x) \to 0$ for $x \to \infty$. So disorder is characterized by two parameters: the variance of the disorder fluctuations, τ_1, and the large, but finite correlation lengthscale of the disorder, R_0. Referring to the latter parameter we will also call the model (1), (2), (5) the large-scale (LS) model. The two parameters can be compared if we define a temperature scale corresponding to R_0 (measured in terms of the lattice spacing a) via

$$\tau_2 = (a/R_0)^2. \tag{6}$$

At this temperature the mean-field thermal correlation length reaches the value $\xi(\tau_2) = a\tau_2^{-1/2} = R_0$.

The influence of thermal fluctuations can be estimated with a third parameter not related to the disorder: the Ginzburg–Levanyuk parameter

$$\tau_G = \left(\frac{\lambda}{a^d}\right)^{2/(4-d)} \tag{7}$$

which limits the critical region to $\tau \leqslant \tau_G$. This is discussed further at the end of section 4.3.

In the following analysis we will see that for different values of the ratio τ_2/τ_1 different results appear. We will start our analysis of possible phase-transition scenarios outside of the critical region. Of course, even in this region, far above the phase-transition temperature of the sample as a whole, nuclei of the low-temperature phase may appear in regions with large, negative values of $\Delta\tau(\boldsymbol{r})$.

3.1. One-instanton approximation

Far above T_c one can calculate the free energy of the system with rather rare, isolated ordered regions using the replica trick for the disorder average which is denoted by $\langle\rangle$:

$$F = -k_B T \langle \ln Z \rangle = -k_B T \lim_{n\to 0} \frac{1}{n}(\langle Z^n \rangle - 1). \tag{8}$$

A cumulant expansion of the terms in $\langle Z^n \rangle$ leads to the familiar replica Hamiltonian for

$$\langle Z^n \rangle = \int \prod_{\alpha=1}^{n} \mathcal{D}\varphi_\alpha \exp(-H_n) \tag{9}$$

$$H_n = \int \mathrm{d}^d \boldsymbol{r} \left(\sum_{\alpha=1}^{n} H_{\mathrm{GL}}[\varphi_\alpha] \right) - \tau_1^2 \int \mathrm{d}^d \boldsymbol{r}\, \mathrm{d}^d \boldsymbol{r}' \sum_{\alpha,\beta=1}^{n} G(\boldsymbol{r}-\boldsymbol{r}')\varphi_\alpha^2(\boldsymbol{r})\varphi_\beta^2(\boldsymbol{r}') \tag{10}$$

where $H_{\mathrm{GL}}[\varphi]$ is given by (2).

Outside of the critical region this functional integral can be evaluated in the saddle-point approximation. The main contribution to F comes from solutions of the saddle-point equation

$$(-\nabla^2 + r_0)\varphi_\alpha(\boldsymbol{r}) + \lambda\varphi_\alpha^3(\boldsymbol{r}) - \tau_1^2 \sum_{\beta=1}^{n} \int \mathrm{d}^d \boldsymbol{r}'\, G(\boldsymbol{r}-\boldsymbol{r}')\varphi_\beta^2(\boldsymbol{r}')\varphi_\alpha(\boldsymbol{r}) = 0. \tag{11}$$

A simpler version of this equation, with $\lambda = 0$, appears in the analysis of the density of states deeply in the gap of a disordered semiconductor [21, 22]. In this case equation (11) is isotropic in replica space, and one can look for solutions

$$\varphi_\alpha(\boldsymbol{r}) = \hat{n}_\alpha \phi(|\boldsymbol{r} - \boldsymbol{r}_0|). \tag{12}$$

Here $\hat{n}_\alpha$ is an arbitrary vector of unit length in replica space and ϕ is a function describing the spatial form of the instanton.

In the case $\tau_2 \gg \tau_1$ (small R_0) two types of solutions can be found [22]:

$$\phi_{JS}(r) \sim \exp(-r^2/l^2) \qquad \text{with } l \approx (R_0\xi)^{1/2} \qquad \text{for } \tau \gg \tau_2 \tag{13}$$

and

$$\phi_L(r) \sim \frac{1}{r}\exp(-r/\xi) \text{ for } \tau \ll \tau_2. \tag{14}$$

Both solutions contain the thermal correlation length $\xi = a/\tau^{-1/2}$. The latter, ϕ_L, is the same as in a system with δ-correlated disorder, [21] while the former, ϕ_{JS}, only appears due to disorder correlations as $G(\boldsymbol{r}) \neq \delta(\boldsymbol{r})$.

The corresponding actions are $H[\varphi_{SJ}] \sim (\tau/\tau_1)^2$ and $H[\varphi_L] \sim (\tau/\tau_s)^{2-d/2}$ with $\tau_s = \tau_1(\tau_1/\tau_2)^{d/(4-d)}$. For very large τ the action is dominated by the saddle-point solution (13); at $\tau \approx \tau_2$ the lengthscales l and ξ as well as the actions $H[\varphi]$ characterizing the solutions (13) and (14) are roughly equal, so a crossover occurs to behaviour dominated by solutions φ_L for $\tau < \tau_2$. Below this crossover temperature τ_2 the system's behaviour coincides with the limiting behaviour for $\tau_2/\tau_1 \to \infty$, i.e. the δ-correlated disorder case (in the electronic problem this leads to the well known Lifshitz form of the tail in the density of states [21]).

For $\tau_2 \gg \tau_1$ the finite scale of the disorder correlations only plays a role above τ_2. However, we will be interested in the opposite limit, $\tau_2 \ll \tau_1$ (i.e. large R_0). Here there is no crossover to the Lifshitz result for δ-correlated disorder, and the appropriate saddle-point solution is φ_{JS} in the whole range of applicability of the one-instanton treatment. This solution explicitly contains the disorder correlation length; it can obviously not be found if the approximation of δ-correlated disorder is used.

Of course, in the doped semiconductor problem the one-instanton approximation is limited to the region far away from the mobility edge, i.e. in our notation to the region $\tau > \tau_1$ in the case $\tau_1 > \tau_2$ and $\tau > \tau_s$ (defined above) in the opposite case $\tau_1 < \tau_2$.

These considerations can be extended to the full equation (11) with a nonzero λ-term if one notes that $\lambda \to 0$ corresponds to the limit $\tau_1, \tau_2 \gg \tau_G$ in the general phase transition problem under consideration. This limit and the breakdown of the one-instanton approach as λ grows larger than a critical value will be discussed in the next section.

3.2. Thermal fluctuations in the one-instanton approach

The introduction of a thermal fluctuation term, $\lambda \neq 0$, introduces a cubic anisotropy (in n-dimensional replica space) into equation (11). For the conventional model with δ-correlated disorder, (1)–(3), this case has been discussed in [15]. Without performing all the elaborate calculations used in [15] in detail we will show how similar results appear in the case of correlated disorder.

In analogy to (12) solutions of (11) can be written as

$$\varphi_B(\boldsymbol{r}) = \hat{n}_\gamma \left(\frac{\tau}{u-\lambda}\right)^{1/2} \chi(x) \tag{15}$$

with $x = r/\xi$ and $\xi = a\tau^{-1/2}$. However, due to the cubic anisotropy $\hat{n}_\gamma$ must now be one of the n unit vectors spanning replica space. In the case of δ-correlated disorder, for large distances, $x \to \infty$, $\chi(x) \sim \frac{1}{x}\exp(-x)$. The action for such an instanton solution will be

$$S[\varphi_B] \sim \frac{(\tau)^{1/2}}{u-\lambda}. \tag{16}$$

Obviously for $\lambda \to 0$ the Lifshitz solution φ_{L} found above is recovered; but (16) shows that for nonzero values of λ the action (i.e. free-energy gain from the creation of an ordered nucleus) $S[\varphi_B]$ can still be large, and one-instanton solutions must be taken into account.

The role of thermal fluctuations in the one-instanton approach can be discussed in great detail following the treatment in [15]. However, here we are only interested in an estimation of a critical value of λ for which the one-instanton approach breaks down because thermal fluctuation become too strong. For this purpose it is enough to check whether the solution of the saddle-point equation is well behaved. For the solution (15) or δ-correlated disorder this leads to the requirement $\lambda < u$; for the case of correlated disorder at $\tau \gg \tau_2$ the

analogous criterion is easily found to be

$$\lambda > \lambda_c \approx \frac{\tau_1^2}{(\tau\tau_2)^{d/4}} = \tau_1^{2-\frac{d}{2}} \left(\frac{\tau_1^2}{\tau\tau_2}\right)^{d/4}. \tag{17}$$

This describes a critical value λ_c which depends on temperature.

In the case $\tau_2 \gg \tau_1$ as $\tau \to \tau_2$ there is a crossover from the one-instanton solution to the δ-correlated disorder approximation as discussed above; at $\tau = \tau_2$ we get $\lambda_c = \lambda_c' = \tau_1^2 \tau_2^{-d/2} = \tau_1^2 R_0^d = u$ in agreement with the above results for δ-correlated case.

In the second possible case, $\tau \gg \tau_1 \gg \tau_2$, there is no crossover to the δ-correlated case, and close to $\tau = \tau_1$ (at which $S[\varphi_{JS}] \approx 1$ and the one-instanton approximation is no longer applicable) one finds

$$\lambda_c = \lambda_c' = \tau_1^{2-\frac{d}{2}} \left(\frac{\tau_1}{\tau_2}\right)^{d/4}. \tag{18}$$

Note that $\lambda_c' \gg \lambda_c'$, so in the case $\tau_1 \gg \tau_2$ the stability of the ordered droplets against thermal fluctuations is larger than in the case $\tau_1 \ll \tau_2$.

For all types of disorder, as long as $\lambda < \lambda_c$ with the appropriate λ_c, thermal fluctuations do not destroy the locally ordered regions that are the basis of a possible percolative transition. Of course, even in this case of stability against thermal fluctuations the one-instanton approach breaks down closer to the transition, so the real character of the transition cannot be established using the one-instanton approximation for the treatment of the replica Hamiltonian.

At the moment there seem to be no conclusive results about the nature of the transition as T_c is approached; both a physical discussion of the mechanism that replaces the homogeneous scenario of the transition driven by thermal fluctuations and explicit quantitative results are lacking. In the next section we will address these issues in the context of a specific model in which they can be discussed and illustrated in a clear and accessible way.

4. The LS cell model

As we have seen in the preceding section, if τ_2/τ_1 is small a new, percolative type of transition may be expected from the one-instanton approximation which applies at temperatures significantly above T_c. We now want to illustrate this transition closer to criticality. To allow a more detailed discussion we start with a specific version of the LS model (1), (2), (5).

We assume that the random potential $\tau(\boldsymbol{r})$ is formed by regions of size R_0 over which τ is constant. These regions or 'cells' are the elementary cells of a superlattice with lattice scale R_0. To eliminate some complications not connected with the general picture of the percolative transition we assume that $\tau(\boldsymbol{r})$ changes its value at the borders between such areas over a distance of the order of the lattice constant a (this requirement will be relaxed in section 4.3). Obviously such a system leads to a correlator as that given in (5). We want to discuss the case

$$\tau_2 \ll \tau_1 \qquad \text{i.e. } R_0 \gg a\tau_1^{-1/2}. \tag{19}$$

Taking the probability distribution $\mathcal{P}[\Delta\tau(\boldsymbol{r})]$ to be Gaussian, $\mathcal{P}[\Delta\tau(\boldsymbol{r})] \propto \exp\{-\frac{1}{2\tau_1^2}\int \mathrm{d}^d r\, \Delta\tau^2(\boldsymbol{r})\}$, simplifies the following considerations. However, they remain valid for a wide class of one-peak distributions with a finite second moment. A Gaussian distribution for the disorder configurations $\{\tau(\boldsymbol{r})\}$ also implies a Gaussian probability

$\tilde{P}(\tau(\boldsymbol{r})) \propto \exp\{-\frac{1}{2\tau_1^2}\Delta\tau^2\}$ that the random temperature at $\boldsymbol{r}$ has the value $\tau(\boldsymbol{r}) = r_0 + \Delta\tau$. This can be rewritten in terms of the 'local transition temperature' $T_c = T_c(\boldsymbol{r}) = T_c^{(0)}(1 - \Delta\tau(\boldsymbol{r}))$ as

$$P(T_c) \propto \exp\left\{-\frac{1}{2\tau_1^2}\left(\frac{T_c^{(0)} - T_c}{T_c^{(0)}}\right)^2\right\}. \tag{20}$$

4.1. Zero approximation in τ_2/τ_1

The cells in which $\tau(\boldsymbol{r})$ is constant can be classified as 'typical' or 'untypical'. 'Typical' cells are characterized by the inequality

$$|\tau(\boldsymbol{r})| \geqslant \tau_2. \tag{21}$$

This implies that the local thermal correlation length ξ corresponding to the reduced temperature within the cell is smaller than the cell itself,

$$\xi(\boldsymbol{r}) = a|\tau(\boldsymbol{r})|^{-1/2} < R_0. \tag{22}$$

So typical cells are far away from (local) 'criticality'; depending on the sign of $\tau(\boldsymbol{r})$ they are in the high-temperature (paramagnetic) phase or the low-temperature (ferromagnetic) phase.

Although one may describe cells with negative τ as locally ordered regions, of course no real phase transition has taken place there (a true phase transition is only possible in an infinite system); the thermodynamic average of the magnetization within these cells will be zero. But most of the time the spins in a typical cell will be aligned parallel to each other. They will only flip to a different direction together and after an activation time that is necessary to overcome the energy barrier between up and down states of the cell—if $\xi < R_0$ a spontaneous thermal fluctuation of range ξ is not sufficient to flip the state of the whole cell; this requires going through an intermediate state with a domain wall separating the cell, and in systems with discrete order parameter the energy cost for such a domain wall is finite (continuous order parameters are discussed in the appendix). 'Nontypical' cells, however, are 'at criticality', as formally $\xi(\boldsymbol{r}) = a|\tau(\boldsymbol{r})|^{-1/2} > R_0$. Here thermal fluctuations extend over the whole cell, so it cannot be considered to be in an ordered state even if $\tau(\boldsymbol{r}) < 0$.

It is easy to check that the majority of cells fulfil (21), justifying the name 'typical'. The probability p_t that a given cell is typical at the temperature T is given by

$$p_t(T) = P(|\tau(\boldsymbol{r})| > \tau_2) = \int_{-\infty}^{-\tau_2} \tilde{P}(\tau)\mathrm{d}\tau + \int_{\tau_2}^{\infty} \tilde{P}(\tau)\,\mathrm{d}\tau. \tag{23}$$

Correspondingly the probability p_u that it is untypical is

$$p_u(T) = \int_{-\tau_2}^{\tau_2} \tilde{P}(\tau)\,\mathrm{d}\tau = \int_{T-\tau_2 T_c^{(0)}}^{T+\tau_2 T_c^{(0)}} P(T_c)\frac{\mathrm{d}T_c}{T_c^{(0)}} \approx 2\tau_2 P(T_c = T). \tag{24}$$

From figure 1 it is easy to see that $p_u/p_t \sim \tau_2/\tau_1$ (see also the argument preceding equation (30)), so at any given temperature $p_u(T) \ll p_t(T)$, and as a 'zero' approximation in the small parameter τ_2/τ_1 the untypical cells can be neglected. Their influence will be discussed further below.

In this approximation the typical cells in the ferromagnetic state form the randomly distributed sites of the standard percolation problem on the superlattice formed by the cells of size R_0. As shown in the appendix, domain walls within typical cells have an exceedingly high free-energy cost. So the order parameter in neighbouring, typical cells with negative

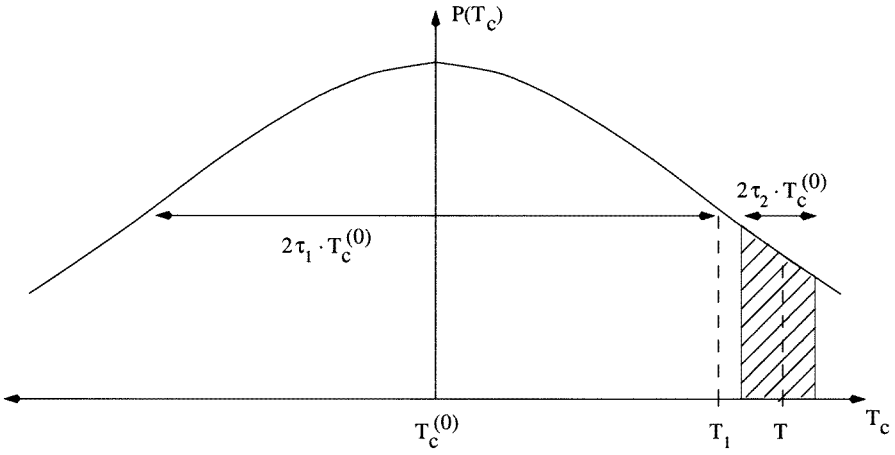

Figure 1. Probability distribution $P(T_c)$ for the local critical temperature $T_c(\boldsymbol{r})$. The shaded area corresponds to the fraction of untypical cells with $|\tau(\boldsymbol{r})| < \tau_2$.

$\tau(\boldsymbol{r})$ must have the same sign, and the clusters of the percolation problem on the superlattice will automatically form magnetically ordered regions, too (of course, the same caveats apply to the expression 'local order' in such clusters as to 'locally ordered' typical cells).

In the limit $\tau_2/\tau_1 \to 0$ the volume fraction of cells in the ferromagnetic state is

$$P_f(T) = \int_{-\infty}^{0} \tilde{P}(\tau)\,\mathrm{d}\tau = \int_{T}^{+\infty} P(T_c)\frac{\mathrm{d}T_c}{T_c^{(0)}}. \tag{25}$$

As T decreases the average size of clusters formed by these ferromagnetic cells, R_p, grows as

$$R_p \sim R_0|P_f(T) - p_c|^{-\nu_p} \tag{26}$$

where R_p is the correlation length, ν_p the correlation length exponent and p_c the percolation concentration of the corresponding percolation problem.

This percolation scenario of the phase transition has first been considered by Ginzburg [23]. He argued that on every cluster of ferromagnetic cells the sign of the order parameter must be constant, as such clusters essentially were 'macroscopic' objects. Thus, Ginzburg considered the temperature T_p corresponding to the (geometric) percolation threshold of the ferromagnetic clusters given by

$$P_f(T_p) = p_c \tag{27}$$

to be the transition temperature at which a nonzero value of the macroscopic magnetization of the sample appears (see figure 2). Correspondingly, the critical exponents are those of the percolation problem.

The percolative character of the transition, which comes from the formation of magnetic domains formed by many locally ordered cells of size R_0, can obviously not be detected in the one-instanton approximation for the replica Hamiltonian presented in the preceding section—as a kind of coherent potential approach the one-instanton approximation can never take into account strong correlations between neighbouring ordered cells.

In this section as a zero approximation we completely neglected the role of the small, but finite fraction of untypical cells. However, it is clear that the qualitative picture of the transition will not be changed by their inclusion—it will still be possible to consider the

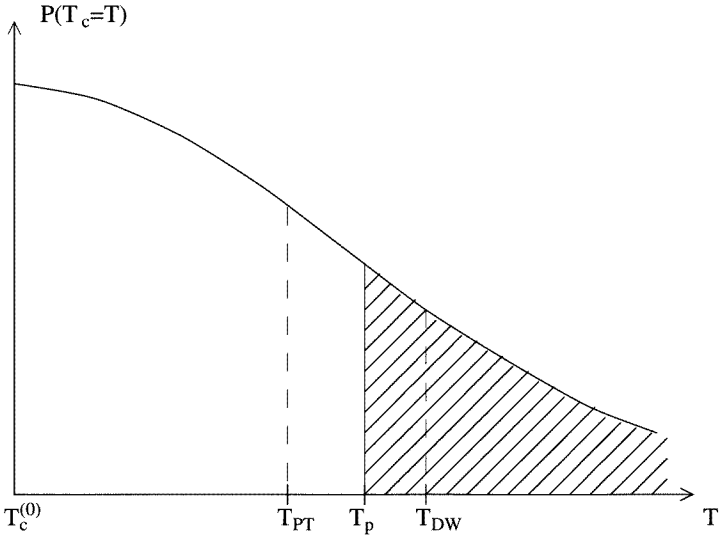

Figure 2. Temperature scale with relevant temperatures and probability that the local critical temperature $T_c(\boldsymbol{r})$ equals T (three-dimensional system, for more details see text): $T_c^{(0)}$ is the transition temperature of the undisturbed system (for $\Delta\tau(\boldsymbol{r}) \equiv 0$). As the temperature drops towards T_{DW} clusters of locally ordered cells begin to form; for $T_{\mathrm{PT}} < T < T_{\mathrm{DW}}$ the distance between broken (untypical) red sites is smaller than the size of locally ordered clusters, so these clusters are split into several distinct ferromagnetically ordered domains; at T_p the regions with $\tau(\boldsymbol{r}) < 0$ percolate; the shaded area corresponds to the critical concentration p_c of the continuous percolation problem of regions with $T_c(\boldsymbol{r}) > T$; T_{PT} is the real phase-transition temperature. At T_{PT} an infinite, monodomain ferromagnetically ordered region appears on the percolating cluster of regions with $\tau(\boldsymbol{r}) < 0$. These temperatures are related as follows: $T_p - T_{\mathrm{PT}} \approx T_{\mathrm{DW}} - T_p \approx \tau_2 T_p$; $T_p - T_c^{(0)} \approx \tau_1 T_c^{(0)}$, where τ_1 is the variance of $P(T_c)$ and τ_2 is defined in equation (7).

formation of macroscopic order as a percolation problem; we will just not be able to treat all cells with negative $\tau(\boldsymbol{r})$ as sites of the percolation problem.

In fact, because in the LS cell model thermal fluctuations enter the picture only via the untypical cells, a careful analysis of their role will allow us to discuss the competition between the quenched disorder, a geometric, temperature-independent property of the system, and thermal fluctuations as macroscopic long-range magnetic order appears in the system.

4.2. Cluster formation and red sites

Stanley, Coniglio and others [24, 25] showed that near the percolation threshold 'red sites' (resp. 'red bonds') play an important role for the geometrical structure of percolation clusters. These sites are located such that removing one of them splits a cluster into two disconnected parts. The number N_1 of such red sites on a cluster with the typical size R_p diverges as [25]

$$N_1 \sim (p - p_c)^{-1}. \tag{28}$$

Following Ginzburg's simplified treatment, the transition occurs when the clusters formed by cells with $\tau(\boldsymbol{r}) < 0$ percolate. Macroscopic order on such a percolation cluster may still be destroyed, however, by the appearance of domains with varying sign of the local

magnetization [26]; the red sites of the percolation cluster are obviously the most likely locations of domain walls between these domains. Although the proportion of red sites that are untypical is small, $p_u \ll p_t$, the probability $p_{\mathrm{DW}} = \exp(-\Delta F)$ for the existence of a domain wall is much larger in an untypical red site, because the free-energy cost ΔF_u for a domain wall in an untypical red site is smaller by a factor of $1/(R\sqrt{\tau_1})^3$ than ΔF_t for a typical red site (this important point is demonstrated in detail in the appendix). So if at a certain temperature locally ordered clusters are split into different domains at all the domain structure will be dominated by domain walls located in untypical red sites.

For our problem both N_1 and p_u are temperature dependent, and one can define a crossover temperature T_{DW} via

$$p_{\mathrm{DW}}(T_{\mathrm{DW}})\,p_u(T_{\mathrm{DW}})N_1(T_{\mathrm{DW}}) \approx 1. \tag{29}$$

At T_{DW} the smallness of p_u is just compensated by the divergence of N_1, while p_{DW} is large and not strongly temperature dependent. For $T > T_{\mathrm{DW}}$ on average there is less than one domain wall per cluster, so the (geometric) clusters of the percolation problem of cells with negative τ can be identified with the (magnetic) clusters of the phase-transition problem that are characterized by having one constant sign of the order parameter over the whole cluster. However, below T_{DW} an increasing number of untypical red sites will contain domain walls. These will divide the geometric clusters into several distinct magnetic domains, and the magnetic correlation length (average size of domains) R_D will be much smaller than the mean cluster size R_p of the geometric percolation problem. In particular, at the temperature T_p defined by (27) the mean value of the magnetization over a macroscopic sample is still zero and the susceptibility finite, in contrast to Ginzburg's prediction [23].

Equation (29) can be used to estimate T_{DW}. Instead of deriving explicit expressions for a particular distribution $P(T_c)$ we will write down a general estimate which holds for all nonsingular, one-peak distributions of the type shown in figure 1.

According to (24) $p_u(T_{\mathrm{DW}}) \sim \tau_2 P(T_{\mathrm{DW}})$. Due to (19) we have $\tau_2 \ll \tau_1$, and the relevant temperatures T_p, T_{DW} and T_{PT} (see below) will all be of the same order as $T_1 = T_c^{(0)}(1+\tau_1)$ (figure 1). Because $\tau_1 T_c^{(0)}$ is the variance of $P(T_c)$ we have $\tau_1 P(T_1) \sim 1$, and the probabilities $P(T)$ for $T = T_1$, T_p, T_{DW} and T_{PT} are all of order $1/\tau_1$. So $p_u(T_{\mathrm{DW}}) \sim \tau_2/\tau_1$. As $P_f(T)$ given in (25) changes slowly for $T \approx T_1$, and using the definition of T_p in (27), we can set $N_1(T_{\mathrm{DW}}) \sim (P_f(T_{\mathrm{DW}}) - p_c)^{-1} \sim (T_{\mathrm{DW}} - T_p)^{-1}$ to find

$$T_{\mathrm{DW}} = T_p(1 + b\tau_2/\tau_1) \tag{30}$$

where b is a numerical factor of order 1.

This crucial influence of untypical red sites on the magnetic structure of the system for temperatures below T_{DW} obviously raises questions about the role of multiply connected sites [27] (which split a cluster into disconnected parts only if all of them are cut simultaneously).

Of course, domains can also be created by cutting multiply connected sites. This possibility can be accounted for generalizing (29) to

$$\sum_{m=1}^{\infty} N_m w_m \approx 1. \tag{31}$$

Here $w_m \sim p_u^m = (\tau_2/\tau_1)^m$ is the probability that all m sites which form one multiply connected site are untypical. N_m is the average number of m-connected sites, which depends on temperature via p in analogy to (28). The probability p_{DW} is not written explicitly because in untypical cells domain walls are very probable, $p_{\mathrm{DW}}^{(u)} \approx 1$, while they do not appear in typical cells (see the appendix). This is also why all of the m sites must be untypical—even one typical site among them would make the free energy cost of the

domain wall as a whole prohibitively large. When condition (31) holds, the structure of the 'geometric' clusters of the percolation problem for ordered cells will significantly differ from the structure of the magnetic domains which is important for the phase transition.

Decreasing the temperature below T_p, the infinite percolating cluster of cells with negative τ (which still contains many disconnected magnetic domains) will grow and form a superlattice with a characteristic lengthscale $R_p(T) \sim (P_f(T) - p_c)^{-\nu_p}$. At a temperature $T_{\rm PT} < T_p$ (figure 2) the volume fraction of *typical* cells with negative τ, which are already in the ferromagnetic state, will reach the percolation concentration p_c. Taking into account untypical cells, the analogue of (25) for small but finite values of τ_2/τ_1 is the volume fraction of typical ferromagnetic cells with $\tau(\boldsymbol{r}) < -\tau_2$:

$$P_f^{(t)}(T) = \int_{-\infty}^{-\tau_2} \tilde{P}(\tau)\,\mathrm{d}\tau = \int_{T+\tau_2 T_c^{(0)}}^{+\infty} P(T_c)\frac{\mathrm{d}T_c}{T_c^{(0)}}. \tag{32}$$

Just as T_p was found from (27) in the 'zero' approximation in the small parameter τ_2/τ_1, now in the first approximation in τ_2/τ_1 the temperature $T_{\rm PT}$ can be determined from

$$P_f^{(t)}(T_{\rm PT}) = p_c \tag{33}$$

so $T_{\rm PT}$ is shifted below the geometric percolation threshold T_p of regions with $\tau(\boldsymbol{r}) < 0$. Obviously in analogy to (30) this shift is proportional to the small parameter τ_2/τ_1,

$$T_p - T_{\rm PT} \approx \tau_2 T_c^{(0)} P(T_c = T_{\rm PT}) \sim \tau_2/\tau_1. \tag{34}$$

At this temperature one infinite domain of ferromagnetic cells with the same value of the order parameter appears, i.e. the magnetization of a macroscopic sample becomes nonzero. This is the real magnetic phase transition point.

It is interesting to note the analogy between the percolative transition in the LS cell model and the transition in the standard diluted Ising model near the $T = 0$, $p = p_c$ bicritical point. The ordered cells of the LS cell model correspond to the occupied sites in the diluted Ising model, and the small probability w_m for a domain wall cutting m multiply connected, untypical sites corresponds to the small probability $\exp(-2mJ/k_BT)$ to break m bonds between Ising spins.

For the diluted Ising model the phase transition line near the bicritical point has been shown to be [28]

$$\frac{1}{p - p_c}\exp(-2J/k_BT) = C. \tag{35}$$

For the two-dimensional bond-diluted Ising model [28] $C = 2\ln 2$. In the general case, the phase transition line is determined in analogy from $\phi(x) =$ constant with an appropriate scaling function ϕ, and the scaling variable given by $x = \frac{1}{p-p_c}\exp(-2J/k_BT)$.

Now in any percolation problem, the average numbers N_m of (m-fold) multiply connected sites on a cluster with the typical size R_p will be of the form

$$N_m(p) \sim \alpha_m(p - p_c)^{-\Delta_m} \tag{36}$$

as $p \to p_c$. Here p_c is the percolation concentration, which depends on the percolation problem under consideration, and p is the volume fraction of occupied sites† i.e. in our case the volume fraction of cells with negative $\tau(\boldsymbol{r})$. For singly and doubly connected bonds it has been shown that [24] $\Delta_1 = 1$, and [27, 30] $\Delta_2 = 2$.

† For some deterministic fractals, which have been used to model typical configurations of the infinite percolating cluster [27], it is easy to check that $\Delta_m = m$ for all m. These fractals are introduced in [29].

With this form of the N_m, condition (31) is an expansion of the scaling function ϕ, allowing a physical intepretation of the terms that appear in ϕ: they come from the various independent ways to produce a domain wall by cutting *multiply* connected sites.

On the other hand, result (35) can only be reproduced from condition (31) if in (36) we have $\Delta_m \leqslant m$ for all m. It is remarkable that this condition concerns the Δ_m, i.e. a purely geometric property of the percolation cluster, although it was derived via the analysis of a thermal phase transition on the percolation cluster.

The role of domain walls is changed qualitatively if we consider models with spin dimension $m > 1$ (like the XY- or Heisenberg model). Here, long-range magnetic order may not only be destroyed by domain walls with a finite width (and energy cost), but also by arbitrarily low-energy extended fluctuations of the order parameter (spin waves). On percolation clusters a transition that breaks a continuous symmetry is generally impossible [31]; if the temperature drops further the infinite cluster has the spatial dimensionality of the imbedding space on lengthscales larger than the connectivity length R_p, so one might expect a transition into an inhomogeneously ordered phase for these models only in the temperature range where $\xi > R_p$. On the other hand, domain walls are as difficult to create as in the Ising model for Pott's models [24] with $q > 2$. One may even speculate whether a crossover to a percolative transition can explain the rounding of the first-order transition in pure Pott's models with $q > 4$ to a continuous transition in the presence of quenched disorder [32].

4.3. Generalization of the LS cell model

In the preceding section the untypical regions where thermal fluctuations may play a significant role, i.e. where $|\tau(\boldsymbol{r})| < \tau_2$, had a minimal scale R_0. In a more natural, smoothly varying random potential $\tau(\boldsymbol{r})$ with correlator (5) this simplification is lost.

Now the disorder correlation scale R_0 still exists, but the disorder *may* also vary over shorter lengthscales. This does not influence the general, percolative nature of the transition or the zero-order analysis presented in section 4.1. The lowest-order influence of the small parameter τ_2/τ_1 is slightly changed, however, because the condition for a spatial region to be considered as 'untypical', which was simply given by (21) before must now be modified.

This can be done in a self-consistent way. We estimate the size l of an untypical region around a point $\boldsymbol{r}_0$ where $\tau(\boldsymbol{r}_0 = 0)$:

$$l(\boldsymbol{r}_0) \leqslant \xi(\boldsymbol{r}_0 \pm l\boldsymbol{e}_i/2) \tag{37}$$

where $\boldsymbol{e}_i$ are unit vectors. In the preceding section it has been shown that for the magnetic-order correlations on locally ordered clusters only untypical regions at red sites (including multiply connected sites) will be significant. The average number of such sites will scale as in (36)—this is a topological property of the percolation problem. However, the probability for such a site to be untypical will be different from the case considered above: untypical red sites now appear at saddle points of the random potential $\tau(\boldsymbol{r})$, so typically the size l_m of an untypical region around a saddle point $\boldsymbol{r}_0$ where $\tau(\boldsymbol{r}_0) = \tau'(\boldsymbol{r}_0) = 0$ (with $\tau' = \mathrm{d}\tau/\mathrm{d}r$) will be given according to (37) by

$$l_m = (\tau' l_m^2)^{-1/2} = (\tau_1\tau_2)^{-1/4} \tag{38}$$

where we have used $\tau' \approx \tau_1/R_0^2$. The dimensionless parameter $\tau_m := 1/l_m^2 = (\tau_1\tau_2)^{1/2}$ can now be used to estimate the fraction $\tilde{p}_u$ of untypical sites as

$$\tilde{p}_u \sim \tau_m/\tau_1 = (\tau_2/\tau_1)^{1/2}. \tag{39}$$

This differs from the result $p_u = \tau_2/\tau_1 \ll \tilde{p}_u$ found above for the LS cell model, but $\tilde{p}_u$ is still a small parameter, and the derivation of T_{DW} presented above remains essentially the same, with $\tilde{p}_u$ replacing p_u.

It is clear that the phase-transition scenario remains percolative in the more general model: the basis for the formation of locally ordered clusters and of domain walls within these clusters is a topological (and universal) property of the underlying percolation problem, namely the distribution of ordered resp. multiply connected sites. Relaxing the requirements for the disorder from a superlattice based structure to a smoothly varying random potential only changes the percolation problem from a lattice to a continuous one.

There is one additional feature in the more general model which has been suppressed in the LS cell model: in the general LS model with smoothly varying 'local T_c' all locally ordered regions have a surface layer (with finite thickness) where $\tau(\boldsymbol{r}) \approx 0$. This surface is at local criticality, so within this surface layer magnetic fluctuation correlations decay very slowly along directions perpendicular to the surface. Parallel to the surface correlations may persist along the whole critical surface.

At the phase-transition temperature T_p the surface is a fractal object that extends through the whole system just as the infinite-ordered cluster itself. Therefore we expect a surface fluctuation contributions to physical quantities such as the specific heat. If the correlation length of thermal fluctuations in the surface layer diverges exactly at T_p such contributions may even be singular, so for a complete description of all thermodynamic properties of the system the surface thermal fluctuations remain to be be accounted for. Moreover, we expect that these fluctuations play an important role in the crossover between the homogeneous and the percolative scenario as the disorder parameters are varied.

So far we have used the mean-field values for critical exponents such as $\nu = \nu_{\mathrm{MF}} = \frac{1}{2}$. Obviously this is possible if the critical region of the pure system is so narrow that $\tau_{Gi} \ll \tau_2, \tau_1$, so all the discussions above refer to temperatures outside of the region of critical thermal fluctuations. We will now discuss briefly the changes that appear when $\tau_2 \ll \tau_{Gi} \ll \tau_1$ or even $\tau_2 \ll \tau_1 \ll \tau_{Gi}$.

As we have seen above, the transition occurs at $\tau = \tau_1(1 - \mathrm{O}(\sqrt{\tau_2/\tau_1}))$. In the case $\tau_2 \ll \tau_{Gi} \ll \tau_1$ for $\tau \approx \tau_1$ the fluctuation contribution is still small and the disorder plays the dominating role. So, up to the phase-transition point the system is not inside the region of critical thermal fluctuations and the mean-field exponents can be used.

In the second case, $\tau_2 \ll \tau_1 \ll \tau_{Gi}$, as $\tau \to \tau_1$ we are inside the critical region. In this case the (local) correlation length of thermal fluctuations will be given by $\xi \sim \tau^{-\nu}$ with a critical exponent $\nu \neq \nu_{\mathrm{MF}}$. Then the definition of the temperature scale τ_2 associated with the disorder dorrelations will have to include the nontrivial exponent, so (6) will be replaced by

$$\tau_2 = (a/R_0)^{1/\nu}. \tag{40}$$

However, this change will only influence the numerical values of quantities that depend on τ_2; as ν will be at least roughly equal to $\frac{1}{2}$, the qualitative picture of the transition will not change, and the results of this section will still hold with τ_2 now defined via (40) instead of (6).

4.4. Failure of the Harris criterion and the standard RG

For the pure Ising model α is positive, and the Harris criterion [9] indicates that the disorder will be relevant. There is no indication of the percolative critical behaviour of the LS model described in the previous section, though. From the Harris criterion alone, such an indication

cannot be expected, as it only demonstrates the inconsistency of a description of the system using the pure critical exponents. Explicit RG calculations would not lead to any indication of the percolative scenario described above, either: the replica trick, which is the usual means of performing the average over disorder configurations, obscures the inhomogeneous nature of the disordered system that is crucial for the percolative transition scenario. In addition, after transformation to lengthscales larger than R_0, the RG will take us back to the standard model (1)–(3) of δ-correlated disorder.

It might be possible to obtain better results using functional RG methods, i.e. not only a renormalization of parameters such as λ and u from (2) and (3), but one that also allows a variation of the functional form of the correlation function $G(r/R_0)$ from (5) under renormalization. However, such techniques have so far only been used successfully in lower-dimensional systems, for example, for an analysis of the wetting problem; for bulk problems, an exact functional renormalization can be formulated, but is very hard to apply in practice [33].

Pure systems with a multicomponent order parameter (XY, Heisenberg etc) have a negative specific heat exponent α, and the Harris criterion (as well as explicit RG calculations) would support the doubts whether a percolative transition can appear in these models that were mentioned in section 4.2. Nevertheless, the Harris criterion does not provide us with a conclusive result confirming or excluding the possibility of a percolative transition, and again the RG cannot be expected to describe such a transition, either.

There is a reason for the failure of the Harris criterion (and the standard RG procedure) to identify critical behaviour of a percolative type: the underlying assumption that there is only one relevant lengthscale in the system, the thermal correlation length. All other lengthscales, including scales characterizing a nontrivial ground state of the system, are deliberately neglected if the disorder is modelled as δ-correlated, and of course the RG procedure by itself does not produce any new lengthscales, either. It does lead to a self-consistent description of the transition, but this simplified picture may not apply to all types of disorder.

In the previous sections, for example, we have demonstrated how disorder with a *finite* characteristic lengthscale R_0 can lead to the appearance of a *diverging* 'geometric' lengthscale in the system, the percolation connectivity length $R_p(T)$ of clusters of ordered regions. So the LS model illustrates a limitation of the standard procedures. For a more careful treatment of disordered systems, one must first take into account geometrical lengthscales associated with the disorder. Then one must check explicitly whether the critical behaviour of the system turns out to be asymptotically independent of such geometrical scales. Only then can one return to the simpler model of δ-correlated disorder; in other cases, as for the LS model considered here, such a simplification is not possible.

5. Summary and discussion

The analysis of the disordered Ising model with two disorder parameters (5), one of them a finite disorder correlation length, revealed a critical behaviour qualitatively different from the usually considered case of δ-correlated disorder. Both in a very specific version of this two-parameter model and in a more general context we showed that nontrivial, inhomogeneous and temperature-dependent ground states have to be accounted for, in contrast to the usual transition scenario driven by thermal fluctuations around a spatially homogeneous ground state.

This inhomogeneous ground state is produced by the appearance of locally ordered regions (nonzero solutions of the nonlinear stochastic Ginzburg–Landau equation). Its

temperature dependence can be discussed in the terms of a percolation phenomenon. The topological properties of the corresponding percolation clusters play a crucial role for the formation of long-range magnetic order and the critical behaviour near the transition: in the immediate vicinity of the real transition point broken untypical red sites of the percolation cluster determine the long-range domain structure of the magnetic order parameter. These broken sites occupy a very small volume fraction of the sample, in fact, not only a small fraction of the ordered regions as a whole, but even a small fraction of all red sites, too. The relevant lengthscale for the transition is not the correlation length ξ of thermal fluctuations (as in the case of δ-correlated disorder) but rather the percolation connectivity length, and, in the immediate vicinity of the critical point, the average size of magnetic domains on the percolation cluster.

This percolative aspect of the phase transition problem cannot be described appropriately within the one-instanton approximation for the replica Hamiltonian which has been the usual tool for the discussion of the influence of nonzero, inhomogeneous ground states far above the transition.

It is interesting to note the qualitative resemblance between some of our results with experimental observations of the magnetic phase transition in [34] and other materials including perovskites [35, 36] (where the transition in the undisturbed system is weakly first order). In these materials scattering experiments revealed the existence of two lengthscales close to T_c. One of them can be interpreted as the usual correlation length ξ of thermal fluctuations. A second, much larger lengthscale is present in addition to ξ. It appears to be related to random elastic stress fields introduced by disorder, e.g. dislocations. These two lengthscales are not observed in macroscopically separated regions of the samples; they have been found to co-exist even in very thin films [37] where the random stress field is due to a lattice mismatch between the film and substrate. As both dislocations and lattice mismatch would introduce disorder with lengthscales much larger than the lattice constant it is tempting to speculate that the second, large lengthscale may be connected with the appearance of locally ordered regions similar to those described in our discussion of the LS model. Percolation of such regions has also been discussed as a mechanism by which the presence of disorder could turn a weakly first-order transition into a continuous one [35].

A different aspect of the existence of nontrivial ground states has been discussed recently [16] within the context of δ-correlated disorder: the fact that there is a macroscopic number of nearly degenerate such ground states (for example, in the Ising case in each ordered region the order parameter can have two different signs). Methods from the theory of spin glasses (where the existence of multiple ground states plays a crucial role) have been used to show the existence of RSB even in the weakly disordered Ising model.

However, in these considerations the temperature dependence and the nontrivial topological percolation structure of the ground states has not entered in any obvious way. Taking into account our results on these properties of the ground states of the LS model one may expect a connection between the value of the Parisi parameter in the RSB mechanism, which is related to the topology of the system's free-energy landscape, and the nontrivial, percolative structure of the order parameter on the locally ordered regions.

The critical behaviour of the LS model is determined by the stochastic geometry of the ferromagnetically ordered clusters. We have demonstrated the crucial role of red sites on these clusters and indicated the possible influence of other, multiply connected sites. In our opinion the importance of the geometrical structure of the percolation cluster for the phase transition in disordered systems warrants further investigation of the statistics of such clusters. So far only limited progress has been made in this respect, [24, 27, 30, 38] although geometric concepts have been applied to study various lattice spin models. With similar

applications in mind we would also hope to encourage a renewed effort to investigate the critical behaviour of spin models on deterministic fractals. On these fractals properties such as ramification and lacunarity and their influence on critical behaviour can be discussed explicitly.

Our discussion of long-range correlated systems with explicit lengthscales of the disorder may be helpful for the understanding of δ-correlated systems and their critical behaviour, as well. In fact, the original lattice versions of these systems may possess their own generic lengthscales of disorder which become evident if one classifies spins as so-called bootstrap spins according to the number z of their nearest neighbours [39, 40]. Clearly, the interesting values of z are between 2 and 6 (in the simple cubic lattice) since only then may one have large clusters and percolation phenomena of bootstrap spins. The corresponding percolation probabilities $p_c^{(z)}$ are in the range $p_c \leqslant p_c^{(z)} \leqslant 1$ with $p_c(z-1) \leqslant p_c^{(z)}$, where p_c is the critical concentration of the conventional percolation problem.

Clusters of bootstrap spins (at least for large enough values of z) are regions of increased local critical temperature $T_c^{(z)} > \langle T_c(\boldsymbol{r})\rangle$. At their percolation concentrations the size of these clusters diverges and the set of bootstrap spins forms a fractal, self-similar part of the system with a critical temperature $T_c^{(z)}$. For such fractals it has been shown [41, 42] that several discrete spin models have phase transitions at finite temperatures and their critical exponents depend on the whole set of fractal characteristics, including the connectivity and lacunarity besides the Haussdorf dimension d_f. The role of bootstrap spin clusters near their percolation point $p_c^{(z)}$ may be similar to that of ferromagnetic cells in the LS model. In both cases geometric lengthscales appear and are concurrent to or even dominate the thermal correlation length.

Bootstrap percolation clusters are also readily identified as regions of the system where a turnover of all spins of the cluster including the boundary spins does not appreciably change the energy but leads to a new, nearly degenerate 'ground state'. In this respect they are equivalent to the locally ordered regions in the LS model discussed above. Thus, a multiplicity of ground states appears to be a generic property of disordered systems at concentrations $p \leqslant p_c(z=6)$ where fractal clusters of bootstrap spins become existent.

Acknowledgments

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Appendix A. Domain wall in an untypical cell

In this appendix we calculate the free-energy cost for placing a domain wall in a red bond of a locally ordered cluster. In one dimension we have to solve the saddle-point equation

$$-\varphi'' + \tau(x)\varphi + \lambda_0\varphi^3 = 0 \tag{A1}$$

with $\varphi'' = \mathrm{d}^2\varphi/\mathrm{d}x^2$. The restriction to one dimension does not change the qualitative results of our considerations, and for large cell diameter R we may recover the general d-dimensional case by multiplying the expressions for the free energy by a factor of R^{d-1}.

If we multiply (A1) by φ' and integrate from $-\infty$ to x we find a 'constant' of integration

$$-\tfrac{1}{2}\varphi'^2 + \tfrac{1}{2}\tau(x)\varphi^2 + \tfrac{1}{4}\lambda\varphi^4 = C + \int_{-\infty}^{x} \tfrac{1}{2}\tau'(x)\varphi^2\,\mathrm{d}x = C + I(x). \tag{A2}$$

Of course, this is only really constant in intervals where $\tau(x)$ does not change.

A.1. Relevant lengthscales

Let us first consider the simple situation where $\tau(x)$ is a step-like function with the value $-\tau_1 < 0$ for $x < 0$ and $0 > -\tau_2 > -\tau_1$ for $x > 0$ (for a start the parameters τ_1 and τ_2 are arbitrary, but in the situation we are most interested in, $\tau_2 \ll \tau_1$ stand for the most probable reduced temperature inside and outside of an untypical cell just as in the main part of this article). Then the exact solution of (A1) is given by [43]

$$\varphi_1 = -\sqrt{\tau_1/\lambda}\tanh(x\sqrt{\tau_1/2} + c_1) \qquad \text{for } x < 0 \tag{A3}$$

$$\varphi_2 = \sqrt{\tau_2/\lambda}\coth(x\sqrt{\tau_2/2} + c_2) \qquad \text{for } x > 0. \tag{A4}$$

The parameters c_1, c_2 are determined requiring continuity of φ at $x = 0$:

$$c_1 = \operatorname{artanh}(-\sqrt{\lambda/\tau_1}\varphi_0) \qquad c_2 = \operatorname{arcoth}(\sqrt{\lambda/\tau_2}\varphi_0). \tag{A5}$$

Here $\varphi_0 = \varphi(0)$, and evaluating (A2) at $x \to \pm\infty$ we obtain

$$-\frac{\tau_1^2}{4\lambda} + \frac{\tau_1 - \tau_2}{2}\varphi_0^2 = -\frac{\tau_2^2}{4\lambda}, \tag{A6}$$

so $\varphi_0 = \sqrt{(\tau_1 + \tau_2)/2\lambda}$.

Obviously for similar values of τ_1 and τ_2 the solution goes to the mean-field value for large $|x|$ on a lengthscale $1/\sqrt{\tau_1}$ resp. $1/\sqrt{\tau_2}$. However, for $\tau_2 \ll \tau_1$ the prefactor of φ_2 given in (A4) is very small while φ_0 is not, so the argument of φ_2 must be small, as well. Then we can expand $\coth(x) \approx 1/x$ to obtain $c_2 \approx \sqrt{2\tau_2/\tau_1}$ and

$$\varphi_2 \approx \sqrt{\tau_1/2\lambda}\frac{1}{1 + x\sqrt{\tau_1}/2}. \tag{A7}$$

So in this case φ_2 drops to $\sqrt{\tau_2/2\lambda}$ slower than exponentially, reflecting the divergence of the lengthscale $1/\sqrt{\tau_2}$ as $\tau_2 \to 0$. However, (A7) still contains the scale $1/\sqrt{\tau_1}$. So φ_2 will become small on a lengthscale $1/\sqrt{\tau_1}$, although not exponentially fast. This fact can be used to obtain a first, qualitative result.

A.2. Qualitative result

A red bond of size R between two locally ordered regions may be described setting $\tau(x) = -\tau_1$ for $|x| > R/2$ and $\tau(x) = -\tau_2$ for $|x| < R/2$.

If the red bond contains a typical cell, τ_1 and τ_2 will be similar. The solution of (A1) will look as shown in figure 3.

Both the antisymmetric solution φ_- with a domain wall and the symmetric solution φ_+ without one reach the mean-field value $\sqrt{\tau_2/\lambda}$ within the red bond rather close to the edge of the red bond, as the relevant lengthscale $1/\sqrt{\tau(x)}$ is small both outside of and within the red bond. So a domain wall within the red cell is essentially a Bloch wall that is a solution of (A1) for constant τ. Its free energy differs from that for the symmetric solution by [43]

$$\Delta F_t = \tfrac{2}{3}\sqrt{2\tau_2^3/\lambda^2}. \tag{A8}$$

However, if the red bond is untypical (and the neighbouring ordered regions are typical, otherwise they would count as part of the red bond), we have $\tau_2 \ll \tau_1$. Then as a first approximation both φ_- and φ_+ will go to (nearly) zero on the (short) lengthscale $1/\sqrt{\tau_1}$.

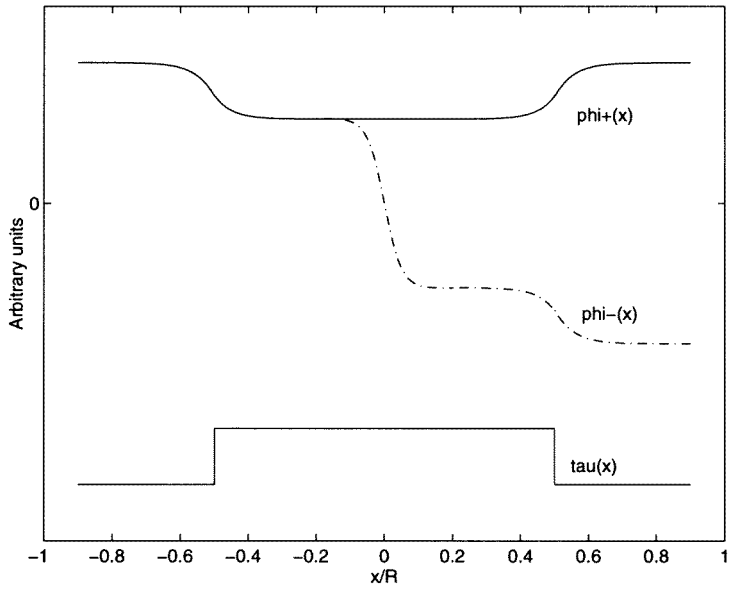

Figure 3. Domain wall in a typical red bond. φ_- is the antisymmetric solution with a domain wall, φ_+ is the symmetric solution without one. The areas where φ is not constant extend over a lengthscale $(\tau(x))^{-1/2}$.

This situation is shown in figure 4. Obviously to this approximation the solutions only differ by the sign for $x > 0$, so they have the same free energy and

$$\Delta F_u \approx 0. \tag{A9}$$

A.3. Exact solution

The above already illustrates the result $\Delta F_u \ll \Delta F_t$ used in the main part of this paper. However, it is also interesting to look at the exact solution of (A1) in an untypical red bond to find the dependence of the small ΔF_u on the parameter $R\sqrt{\tau_1}$ which is large because τ_1 describes a typical cell according to (21).

To this purpose we briefly present the exact solution of (A1) for an untypical red cell, omitting most of the details of the calculations. As the lengthscale $1/\sqrt{\tau_2}$ is then irrelevant anyway we can simplify the calculations by setting $\tau_2 = 0$ and writing τ instead of τ_1.

For $x < -R/2$ we obtain, just as in the simpler case considered above,

$$\varphi_\pm = -\sqrt{\tau/\lambda}\tanh(x\sqrt{\tau/2} + c_\pm) \tag{A10}$$

with $c_\pm = \operatorname{artanh}(\varphi_{0\pm}\sqrt{\lambda/\tau})$, where $\varphi_{0\pm} = \varphi_\pm(-R/2)$ is to be determined later. For $x > R/2$ the same solution appears, with signs that can be read off from figure 4.

In intervals where $\tau(x)$ (and, consequently, $I(x)$) is constant one can rearrange (A2) to obtain an expression for $\mathrm{d}\varphi/\mathrm{d}x$ and separate variables to obtain the integral equation

$$x - x_0 = \int_{\varphi(x_0)}^{\varphi(x)} \frac{\mathrm{d}\varphi}{\sqrt{\tau(x)\varphi^2 + \frac{\lambda}{2}\varphi^4 - 2(C + I)}}. \tag{A11}$$

Inside the red bond the sign of $C + I_\pm$ can be determined from (A2) at $x = 0$ as

$$C + I_+ = \lambda\varphi_m^4/4 > 0 \tag{A12}$$

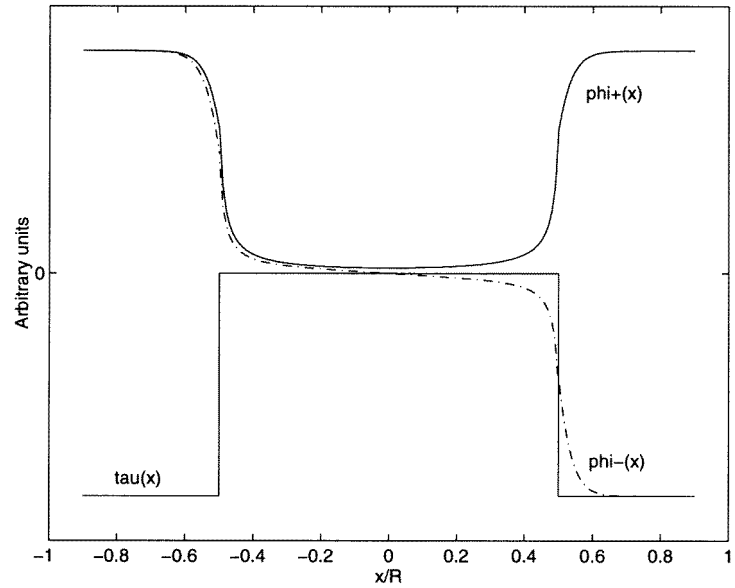

Figure 4. Domain wall in an untypical red bond. As a first approximation the two solutions only differ by their sign for $x > 0$. The only relevant lengthscale is that connected with the reduced temperature outside of the red bond, $(\tau(x \to \infty))^{-1/2}$.

$$C + I_- = -\varphi'_-(0)^2/2 < 0. \tag{A13}$$

Here $\varphi_m = \varphi_+(0)$ is the minimum value of φ_+ within the red bond.

Expression (A11) with $\tau(x) = 0$ then leads to elliptic integrals which can be solved with appropriate substitutions [44] for φ to obtain solutions in terms of the Jacobian elliptic function $\mathrm{nc}(x|\frac{1}{2})$. For the sake of brevity we write $\mathrm{nc}(x)$ instead of $\mathrm{nc}(x|\frac{1}{2})$. Then

$$\varphi_+(x) = \varphi_m \mathrm{nc}(z_+(x)) \tag{A14}$$

$$\varphi_-(x) = B\left(\frac{\mathrm{nc}(z_-(x)) - 1}{\mathrm{nc}(z_-(x)) + 1}\right)^{1/2}. \tag{A15}$$

The parameters are $\varphi_m = (4(C + I_+)/\lambda)^{1/4}$ and $B = (-4(C + I_-)/\lambda)^{1/4}$, and $z_\pm(x)$ are shorthand for the arguments of nc, $z_+(x) = x\varphi_m\sqrt{\lambda}$ in φ_+ and $z_-(x) = xB\sqrt{2\lambda}$ in φ_-.

The parameters are related to the value of $\varphi_{0\pm}$ at the edge of the red bond via $C + I_\pm$:

$$\lambda\varphi_m^4/4 = C + I_+ = -\frac{\tau^2}{2\lambda} + \tau\varphi_{0+}^2 \tag{A16}$$

$$-\lambda B^4/4 = C + I_- = -\frac{\tau^2}{2\lambda} + \tau\varphi_{0-}^2. \tag{A17}$$

In terms of $z_\pm = z_\pm(-R/2)$ this yields

$$\bar{\varphi}_m = -\frac{2z_+}{R\sqrt{\tau}}; \bar{\varphi}_{0+} = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{2z_+}{R\sqrt{\tau}}\right)^4 + 1} \tag{A18}$$

$$\bar{B} = -\frac{\sqrt{2}z_-}{R\sqrt{\tau}}; \bar{\varphi}_{0-} = \frac{1}{\sqrt{2}}\sqrt{1 - \left(\frac{\sqrt{2}z_-}{R\sqrt{\tau}}\right)^4}. \tag{A19}$$

The bar denotes normalization with respect to $\varphi_\pm(x \to -\infty) = \sqrt{\tau/\lambda}$, so $\bar{\varphi} = \sqrt{\lambda/\tau}\varphi$ etc.

We can use these expressions to write the squares of the solutions (A14) and (A15) at $x = -R/2$,

$$\left(\frac{2z_+}{R\sqrt{\tau}}\right)^2 + \left(\frac{R\sqrt{\tau}}{2z_+}\right)^2 = 2\mathrm{nc}^2(z_+) \tag{A20}$$

$$\left(\frac{R\sqrt{\tau}}{\sqrt{2}z_-}\right)^2 - \left(\frac{\sqrt{2}z_-}{R\sqrt{\tau}}\right)^2 = 2\frac{\mathrm{nc}(z_-) - 1}{\mathrm{nc}(z_-) + 1}. \tag{A21}$$

These equations must be solved numerically to find $z_\pm$ and the other parameters. However, as $R\sqrt{\tau}$ is large the solutions will be close to the singularity of the expressions on the right hand side of (A20) and (A21), so it is clear that $z_+ \approx 1.8$ and $z_- \approx 3.6$.

Using these values we obtain

$$\bar{\varphi}_{0+}^2 - \bar{\varphi}_{0-}^2 = \frac{1}{2}\left(\frac{\sqrt{2}}{R\sqrt{\tau}}\right)^4 (4z_+^4 + z_-^4). \tag{A22}$$

So the difference between the two solutions is small at $x = -R/2$ and they are both close to the solution found from (A6) for the simpler case of one step in $\tau(x)$, i.e. $\bar{\varphi}_{0+} \approx \bar{\varphi}_{0-} \approx 1/\sqrt{2}$. This implies $\bar{\varphi}_{0+} - \bar{\varphi}_{0-} \approx (\bar{\varphi}_{0+}^2 - \bar{\varphi}_{0-}^2)/\sqrt{2}$ and $\bar{\varphi}_{0+}^3 - \bar{\varphi}_{0-}^3 \approx 3(\bar{\varphi}_{0+}^2 - \bar{\varphi}_{0-}^2)/2\sqrt{2}$. These results are needed in the next section where we calculate the free-energy difference between the two solutions.

A.4. Free energy

The expression for the free energy of a given solution of (A1) can be simplified using (A2):

$$F[\varphi] = \int_{-\infty}^{\infty} (\varphi'^2 + C + I(x))\,\mathrm{d}x. \tag{A23}$$

If we can express the derivative φ' in terms of the original solution φ this simplifies further, as

$$\int_{x_1}^{x_2} \varphi'^2\,\mathrm{d}x = \int_{\varphi(x_1)}^{\varphi(x_2)} \varphi'(\varphi)\,\mathrm{d}\varphi. \tag{A24}$$

There are three contributions to the free-energy difference $\Delta F = F_- - F_+$. The first comes from the intervals $|x| > R/2$. There φ_- and φ_+ have the same functional form, but are shifted apart by $c_- - c_+$ in the x-direction. We note that $\varphi' \propto 1/\cosh^2 \propto (1 - \tanh^2)$ to apply (A24) and obtain, using the results from the preceding section,

$$\Delta F_1 = 2\int_{\varphi(c_+)}^{\varphi(c_-)} \varphi'(\varphi)\,\mathrm{d}\varphi \tag{A25}$$

$$= \sqrt{2\tau^3/\lambda^2}(\bar{\varphi}_{0+} - \bar{\varphi}_{0-}) - (\bar{\varphi}_{0+}^3 - \bar{\varphi}_{0-}^3)/3 \tag{A26}$$

$$= \sqrt{2\tau^3/\lambda^2}\frac{4z_+^4 + z_-^4}{4\sqrt{2}(R\sqrt{\tau})^4}. \tag{A27}$$

In these intervals $I(x)$ is the same for both solutions, so there is no contribution to ΔF_1 from $\int (C + I)\,\mathrm{d}x$.

Within the red cell $I_- \neq I_+$, so there is a contribution

$$\Delta F_2 = R(I_- - I_+) = \frac{R\sqrt{\tau}}{\sqrt{2}}\sqrt{2\tau^3/\lambda^2}(\bar{\varphi}_{0-}^2 - \bar{\varphi}_{0+}^2)/2 \tag{A28}$$

$$= -\sqrt{2\tau^3/\lambda^2}\frac{4z_+^4 + z_-^4}{4(R\sqrt{\tau})^3}. \tag{A29}$$

The last contribution comes from the difference between the solutions within the red cell. Using the relations between the Jacobian elliptic functions nc, sc and dc [44] it is easy to write $\varphi'_\pm$ as functions of $\varphi_\pm$ and use (A24). After some algebra and the integration we obtain as a final result

$$\Delta F_3 = \int_{-R/2}^{R/2} (\varphi'^2_- - \varphi'^2_+)\,\mathrm{d}x \tag{A30}$$

$$= \sqrt{2\tau^3/\lambda^2}\left(\frac{4z_+^4 + z_-^4}{2\sqrt{2}(R\sqrt{\tau})^4} + 2\frac{4z_+^4 - z_-^4}{3(R\sqrt{\tau})^3}\right). \tag{A31}$$

Thus, the leading contributions to ΔF are $\mathrm{O}(1/(R\sqrt{\tau})^3)$ and come from ΔF_2 and the second term in ΔF_3. They are both connected by the fact that the solutions φ_- and φ_+ differ outside and at the edge of the red bond. This is remarkable because the 'obvious' way to estimate ΔF would be to focus on the contribution from $\int(\varphi'^2_- - \varphi'^2_+)\,\mathrm{d}x$ within the cell. In the exact solution this term only contributes a small correction $\mathrm{O}(1/(R\sqrt{\tau})^4)$. In addition, this latter contribution calculated with a simple linear superposition ansatz $\varphi_\pm = \varphi_1 \pm \varphi_2$ with suitably chosen φ_1 and φ_2, for example, using (A7) yields $\Delta F \propto 1/R^2$ which is different from our exact result. Such a superposition ansatz also overestimates the difference $\varphi_{0+} - \varphi_{0-}$ of the solutions at the edge of the red bond to be $\mathrm{O}(1/R)$ instead of $\mathrm{O}(1/(R\sqrt{\tau})^4)$ in our exact result.

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